

A finite layer technique for calculating three-dimensional pollutant migration in soil

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A technique for the analysis of two- and three-dimensional pollutant migration through a layered soil medium is described. An earlier solution for plane diffusion in a single homogeneous layer of soil is extended using the finite layer method for general three-dimensional diffusion. Particular attention is focused on the effects of horizontal advective velocity and coefficient of hydrodynamic dispersion within the aquifer together with the thickness of the aquifer. A parametric study is presented to demonstrate some characteristics of contaminant migration in a layered soil system, taking into account the fact that the surface concentration does not remain constant because of contaminant transport into the deposit. The advantages of the approach are most pronounced when attempting to determine concentrations away from the landfill at modest to large times.

Dans cet article une technique d'analyse bidimensionnelle et tridimensionnelle de la migration des pollutions à travers un sol multicouches est décrit. Une méthode pour la diffusion plane dans une seule couche homogène de sol est développée à l'aide de la méthode des couches fines pour la diffusion tridimensionnelle générale. L'attention est portée plus particulièrement sur les effets de la vitesse d'advection horizontale et au coefficient de dispersion hydrodynamique à l'intérieur de l'aquifère et à son épaisseur. Une étude des paramètres est présentée pour démontrer quelques caractéristiques de la migration des pollutions dans un sol multicouches, en égard au fait que la concentration superficielle ne reste pas constante, en raison du transfert des pollutions dans le dépôt. L'avantage principal de la méthode consiste en ce qu'elle aide à la détermination des concentrations à long ou à court terme.

KEYWORDS: analysis; clays; computation; groundwater; industrial waste; water flow.

NOTATION

A_{LF}	average plan area of the landfill
c	concentration of contaminant
c_A, c_0	initial concentration of contaminant within the landfill

c_b	concentration of contaminant within an aquifer
$c_{b \max}$	maximum concentration ever reached at a particular point in the aquifer
c_{LF}	concentration of contaminant within the landfill
D	isotropic coefficient of hydrodynamic dispersion
D_{Hi}	coefficient of hydrodynamic dispersion in the horizontal plane for aquifer i
D_{vi}	coefficient of hydrodynamic dispersion in the vertical plane for aquifer i
D_{xx}, D_{yy}, D_{zz}	coefficient of hydrodynamic dispersion (including diffusion and mechanical dispersion) in the three Cartesian directions
f	mass flux
f_b	mass flux into base aquifer
f_{LF}	mass flux from the landfill into the soil
h_i	thickness of aquifer i
H_t	height of leachate (volume of leachate per unit plan area)
H_i	thickness of clay layer i
K	distribution coefficient
L	width of the landfill parallel to the velocity v_b
n	porosity
n_b	porosity of the base aquifer
v	seepage velocity
v_a	apparent (Darcy, superficial) velocity
v_{bi}	apparent (Darcy, superficial) velocity in aquifer i
z	distance below the base of the landfill
ρ	bulk density of the soil solids

INTRODUCTION

The design and subsequent environmental approval of landfill sites involves ensuring that the groundwater in regions outside the boundaries of the landfill (and any associated buffer zones) will not be contaminated. In effect, this

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means limiting the concentration of contaminant species in the groundwater to acceptable levels. Regulatory authorities often require the use of a liner which is designed to minimize contamination of the surrounding groundwater. Frequently, these liners take the form of a natural deposit of clay or a compacted clay liner.

The movement of pollutants through relatively impermeable clayey soils is quite slow; however, it is conceivable that significant pollution might occur in the long term and so it is important that such disposal sites should be designed to prevent the possible contamination of the groundwater system in both the short and the long term.

The main factors which govern contaminant migration are advection, dispersion, diffusion and chemical reaction. For many applications, the advective transport is small compared with that due to dispersion (e.g. Crooks & Quigley, 1984). This situation has been investigated using both analytic and numerical techniques (see Freeze & Cherry (1979) for the general background and Anderson (1979) for a detailed review). However, existing solutions have not considered a number of important practical situations.

More recently, Rowe & Booker (1985) have developed a one-dimensional analysis which takes account of the fact that the concentration of leachate in the landfill will decrease as leachate is transported into the clay and which allows for the possible presence of a more permeable underlying stratum (aquifer) beneath the clay deposit. This analysis has been extended by Rowe & Booker (1984a) to include the effects of several different soil layers.

The one-dimensional analysis may be adequate provided that the width of the landfill is large compared with the depth of the clay layer and transport is predominantly vertical. If, however, the plan dimensions of the landfill are comparable with the depth of the layer both horizontal and vertical transport may occur.

In a recent paper, Rowe & Booker (1984b) have developed a solution for plane diffusion in a single homogeneous layer of soil. In this Paper, this approach is extended, using the finite layer method (Cheung, 1976; Rowe & Booker, 1982; Booker & Small, 1982a, 1982b) to account for general three-dimensional diffusion in a horizontally layered soil deposit. A limited parametric study will then be presented to demonstrate some of the important characteristics of contaminant migration in a layered soil system.

THEORY: GOVERNING EQUATIONS

The transport of substances through a saturated clay can often be approximated by a Fickian-

type law (e.g. Gillham & Cherry, 1982), having the form

$$f = nc\nu - nM_D \nabla c \quad (1)$$

$$M_D = \text{diag} (D_{xx}, D_{yy}, D_{zz})$$

where f is the flux and c the concentration of the dispersing substance, n is the effective porosity of the clay, M_D is the matrix of 'coefficients of hydrodynamic dispersion' (incorporating the effects of molecular diffusion and mechanical dispersion) and ν is the seepage velocity. The velocity $\nu_a = n\nu$ will be used to describe advection when discussing the results. The quantity ν_a is variously called the advective velocity, the superficial velocity, the apparent velocity or the discharge velocity. For simplicity of presentation, the term advective velocity will be used throughout this Paper.

Consideration of mass balance shows that

$$\nabla^T f + n \frac{\partial c}{\partial t} + g = 0$$

where the quantity g takes account of the possibility of some of the contaminant being adsorbed on to the clay skeleton. For equilibrium-controlled ion exchange where the concentration of the exchange ion is relatively low, the adsorption of this species may be approximated by a linear relationship of the form

$$g = \rho K \frac{\partial c}{\partial t} \quad (2)$$

where ρ is the bulk density of solid and K is the distribution coefficient. The distribution coefficient K may often be estimated from the results of a laboratory column test (Rowe, Caers, Booker & Crooks, 1984) or may be determined independently (Griffin, Cartwright, Shimp, Steele, Ruch, White, Hughes & Gilkeson, 1976). It should be determined over a representative range of concentrations which reflect the likely field variation.

For an isotropic homogeneous layer in which the pore fluid velocity is uniform, equations (1) and (2) can be combined to give

$$\nabla^T (M_D \nabla c) - \nu^T \nabla c = \left(1 + \frac{\rho K}{n}\right) \frac{\partial c}{\partial t} \quad (3)$$

SOLUTION USING INTEGRAL TRANSFORMS

The transport process is governed by equations (1)–(3) together with the boundary conditions and the condition that any initial background concentration is in equilibrium. For simplicity of presentation, attention will be restricted to the case in which the seepage velocity throughout a particu-

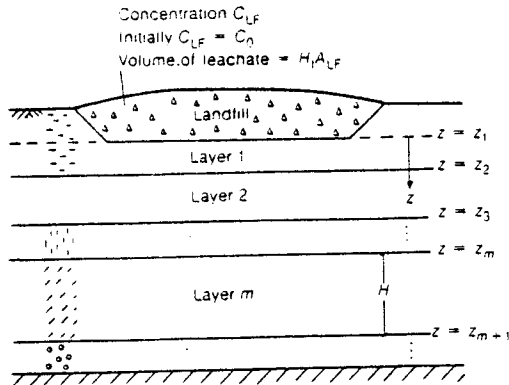


Fig. 1. Cross-section through a landfill with average plan area A_{LF}

lar layer is uniform and in the vertical direction so that

$$v = \left(0, 0, \frac{v_z}{n} \right)^T$$

(Extension of the solution for flow in the x and y directions is straightforward.)

Suppose now that the soil deposit is horizontally layered as shown in Fig. 1. In a typical layer m , $z_m \leq z \leq z_p$ ($p = m + 1$), equations (1)–(3) can be simplified by introducing a Laplace transform

$$(\bar{c}, \bar{f}) = \int_0^\infty (c, f) \exp(-st) dt \quad (4)$$

and a repeated Fourier transform

$$(C, F) = \frac{1}{4\pi^2} \int_{-\infty}^\infty \int_{-\infty}^\infty (c, f) \times \exp[-i(\xi x + \eta y)] dx dy \quad (5)$$

The Fourier integral theorem shows that equation (5) is equivalent to the representation

$$(c, f) = \int_{-\infty}^\infty \int_{-\infty}^\infty (C, F) \times \exp[i(\xi x + \eta y)] d\xi d\eta \quad (6)$$

When the transforms defined by equations (4) and (5) are applied to equations (1) and (3)

$$F_x = -i\xi n D_{xx} C \quad (7a)$$

$$F_y = -i\eta n D_{yy} C \quad (7b)$$

$$F_z = v_z C - n D_{zz} \frac{\partial C}{\partial z} \quad (7c)$$

$$D_{zz} \frac{\partial^2 \bar{C}}{\partial z^2} - (\xi^2 D_{xx} + \eta^2 D_{yy}) \bar{C} - \frac{v_z}{n} \frac{\partial \bar{C}}{\partial z} = \left(1 + \frac{\rho K}{n} \right) s \bar{C} \quad (8)$$

where the properties n , D_{xx} , D_{yy} , D_{zz} , ρ and K are those of the m th layer.

For a local set of co-ordinates with origin at the top of layer m , it is a simple matter to show that equation (8) has the solution

$$\bar{C} = \bar{C}_m \frac{\exp(\alpha Z + \beta) - \exp(\beta Z + \alpha)}{\exp \beta - \exp \alpha} + \bar{C}_p \frac{\exp(\beta Z) - \exp(\alpha Z)}{\exp \beta - \exp \alpha} \quad (9)$$

where \bar{C}_p and \bar{C}_m denote the transformed concentrations on the node planes $z = z_p$ and $z = z_m$ respectively, $H = z_p - z_m$ is the depth of the layer, $Z = z/H$, and $\lambda = \alpha$ and $\lambda = \beta$ are the roots of the equation

$$\lambda^2 - 2P\lambda + Q = 0 \quad (10)$$

with

$$P = \frac{v_z H}{2n D_{zz}}$$

$$Q = \frac{\xi^2 H^2 D_{xx} + \eta^2 H^2 D_{yy}}{D_{zz}} + \left(1 + \frac{\rho K}{n} \right) \frac{s H^2}{D_{zz}}$$

so that

$$\alpha = P + (P^2 - Q)^{1/2}$$

$$\beta = P - (P^2 - Q)^{1/2}$$

where again the properties H , n , D_{xx} , D_{yy} , D_{zz} , ρ and K refer to those of the m th layer.

LAYER MATRIX

The transformed components of vertical flux \bar{F}_p and \bar{F}_m for the node planes $z = z_p$ and $z = z_m$ respectively may be calculated using equations (7)–(9) and it is found that

$$\begin{bmatrix} \bar{F}_m \\ -\bar{F}_p \end{bmatrix} = \begin{bmatrix} S_{11}^{(m)} & S_{12}^{(m)} \\ S_{21}^{(m)} & S_{22}^{(m)} \end{bmatrix} \begin{bmatrix} \bar{C}_p \\ \bar{C}_m \end{bmatrix} \quad (11)$$

where

$$S_{11}^{(m)} = \frac{n D_{zz}}{H} \frac{\beta \exp \beta - \alpha \exp \alpha}{\exp \beta - \exp \alpha}$$

$$S_{12}^{(m)} = \frac{n D_{zz}}{H} \frac{\alpha - \beta}{\exp \beta - \exp \alpha}$$

$$S_{21}^{(m)} = \frac{n D_{zz}}{H} \frac{(\alpha - \beta) \exp(\alpha + \beta)}{\exp \beta - \exp \alpha}$$

$$S_{22}^{(m)} = \frac{n D_{zz}}{H} \frac{\alpha \exp \beta - \beta \exp \alpha}{\exp \beta - \exp \alpha}$$

The matrix S in equation (11) is called the layer matrix for layer m .

Assembly of layer matrices

The layer matrix (11) can be used to solve the general problem for a layered system. To do this, at the interface of adjacent layers, the concentration will be continuous and the net vertical flux entering that particular node plane will be zero. Thus

$$S\bar{C} = F \tag{12}$$

where $C = (C_0, C_1, \dots, C_N)^T$ is the vector of transformed nodal concentrations, $F = (F_0, 0, \dots, -F_N)^T$ is the vector of transformed boundary fluxes and

$$S = \begin{bmatrix} S_{11}^{(0)} & S_{12}^{(0)} & 0 & \dots \\ S_{21}^{(0)} & S_{22}^{(0)} + S_{11}^{(1)} & S_{12}^{(1)} & \dots \\ 0 & S_{21}^{(1)} & S_{22}^{(1)} + S_{11}^{(2)} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Equation (12) has to be modified to incorporate the boundary conditions.

Often, it is found that a given layered deposit will be underlain by a thin more permeable stratum of thickness h in which the groundwater flows with a uniform velocity v_b in the x direction. If it is assumed that the concentration in this stratum is virtually uniform across its width, it then follows from a consideration of conservation of mass that

$$c_b(x, y, t) = \int_0^t \left(\frac{f_b}{n_b h} - \frac{v_b}{n_b} \frac{\partial c_b}{\partial x} + D_{xxb} \frac{\partial^2 c_b}{\partial x^2} + D_{yyb} \frac{\partial^2 c_b}{\partial y^2} \right) dt \tag{13}$$

where $c_b(x, y, t)$ denotes the concentration in the underlying stratum, $f_b = f_N$ denotes the flux entering the base stratum from the clay layer and the subscript b is used to indicate properties of the base stratum.

If equation (13) is transformed using equations (4) and (5), it is found that

$$\bar{F}_N = \chi \bar{C}_N$$

where

$$\chi = h(n_b s + i\xi v_b + n_b D_{xxb} \xi^2 + n_b D_{yyb} \eta^2) \tag{14}$$

Suppose now, for simplicity, that the surface concentration c_0 is specified; then equations (12) and (14) become

$$S^* \bar{C}^* = F^* \tag{15}$$

where $\bar{C}^* = (\bar{C}_1, \bar{C}_2, \dots, \bar{C}_N)^T$ is the vector of unknown transformed concentrations, $F^* = (-S_{21}^{(0)} \bar{C}_0, 0, \dots, 0)^T$ and

$$S^* = \begin{bmatrix} S_{22}^{(0)} + S_{11}^{(1)} & S_{12}^{(1)} & 0 \\ S_{21}^{(1)} & S_{22}^{(1)} + S_{11}^{(2)} & S_{12}^{(2)} \\ \vdots & \vdots & \vdots \\ 0 & 0 & \vdots \\ \vdots & \vdots & \vdots \\ 0 & 0 & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ S_{22}^{(N-1)} + S_{11}^{(N)} & S_{12}^{(N)} \\ S_{21}^{(N)} & S_{22}^{(N)} + \chi \end{bmatrix}$$

Equation (15) can be used to determine $\bar{C}_1, \dots, \bar{C}_N$ and hence, using equation (9), the complete distribution of \bar{C} . This represents the solution in transform space. The Fourier transforms can be inverted by numerical quadrature of equation (6) while the Laplace transforms can be inverted by an efficient algorithm developed by Talbot (1979). This then completes the solution.

ANALYSIS WHEN SURFACE CONCENTRATION VARIES WITH TIME

In many practical situations the surface concentration of the landfill does not remain constant but will vary because contaminant is being transported into the layered soil deposit.

To analyse this situation, consider a landfill occupying a region of plan area A_{LF} to an equivalent height H_f (where H_f represents the volume of leachate divided by the average plan area of the landfill). For simplicity, it is assumed that the concentration of leachate in the landfill does not vary throughout the landfill but may vary with time.

Now suppose that the total mass of landfill that has been placed up to time t is $M(t)$; then from a consideration of conservation of mass

$$A_{LF} H_f c_{LF}(t) = M(t) - \int_0^t \left(\int_{A_{LF}} f_{LF}(x, y, \tau) dx dy \right) d\tau \tag{16}$$

where A_{LF} is the area of landfill, $f_{LF}(x, y, \tau)$ is the flux leaving the landfill at the point (x, y) at time $t = \tau$.

Taking the Laplace transform of equation (16), it is found that

$$\bar{c}_{LF} = \bar{c}_A - \frac{1}{s A_{LF} H_f} \int_{A_{LF}} \bar{f}_{LF}(x, y) dx dy \tag{17}$$

If equation (17) is solved in terms of the yet unknown surface concentration c_{LF} and this solu-

tion is used to calculate the transformed surface flux

$$\bar{F}_{LF} = \Psi \bar{C}_{LF} \quad (18)$$

where Ψ is a known function of ζ and η and

$$\bar{C}_{LF} = \bar{c}_{LF} T$$

with

$$T = \frac{1}{4\pi^2} \iint_{A_{LF}} \exp[-i(\zeta x + \eta y)] dx dy$$

Now

$$f_{LF} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[i(\zeta x + \eta y)] F_{LF} d\zeta d\eta$$

and so substituting into equation (18)

$$\bar{c}_{LF} = \frac{\bar{c}_A}{1 + \Lambda} \quad (19)$$

where

$$\Lambda = \frac{4\pi^2}{sA_{LF}H_t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T^* T \Psi d\zeta d\eta$$

and the asterisk denotes the complex conjugate.

Equation (19) enables the surface concentration c_{LF} to be determined and so the problem may be solved as outlined in the previous section.

RESULTS

The theory described in the previous section is applicable for a wide range of practical problems involving three-dimensional advective-dispersive transport through soil. The major computational effort involved is associated with the numerical inversion of the Laplace and Fourier transforms. Some care needs to be exercised to perform these inversions efficiently and accurately; however, the approach has significant advantages over alternative finite element and finite difference approaches in that it involves minimal data preparation and much less computational effort to obtain results of equivalent accuracy. The advantages of the proposed approach are most pronounced when it is attempted to determine concentrations away from the landfill at modest to large times (although the proposed procedure is computationally superior and easier to use for all the cases examined by the Authors).

The computational effort in this analysis, as with any other three-dimensional analysis, is quite large and makes the performance of a parametric study prohibitively expensive. However, the general effect of the parameters which would influence a full three-dimensional analysis can be illustrated by considering a typical two-

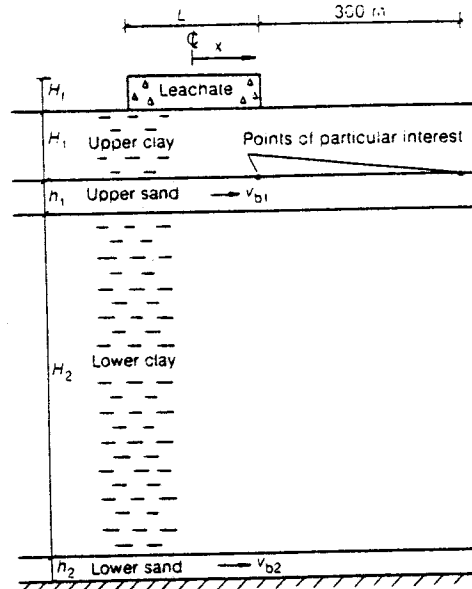


Fig. 2. Problem analysed (the parameters considered are listed in Table 1)

dimensional cross-section such as that illustrated in Fig. 2. This figure shows a stratigraphy similar to that commonly encountered in areas subjected to recent glaciation. Here the landfill is separated from a relatively thin underlying aquifer by a layer of clay or clayey till. The aquifer is underlain by a second thicker layer of clay or clayey till which in turn is underlain by a second sand layer.

In such a situation, the designer is concerned with the concentration of contaminant in the aquifers (particularly the upper aquifer) at the edge of the landfill and at the boundary of the property. Specifically it is assumed (unless otherwise noted) that the deposit has parameters given in Table 1 for a landfill of length 200 m where it is further assumed that the boundary of the property is 300 m from the edge of the landfill (i.e. 400 m from the centre of the landfill).

In the analysis of problems similar to that shown in Fig. 2 (either in two or three dimensions) the major uncertainty is usually associated with the properties of the aquifer. In particular, the advective velocity v_b and the horizontal and vertical dispersion coefficients in the aquifer (D_H and D_v) may be difficult to determine and the effect of a reasonable range of these parameters should be considered.

Figure 3 shows the effect of the horizontal coefficient of hydrodynamic dispersion D_{H1} on the contaminant plume within the upper aquifer at a time 300 years after construction of the landfill (given $D_v = 0.2 \text{ m}^2/\text{year}$, $v_b = 1 \text{ m/year}$). For a

Table 1. Parameters considered in the study

Layer	Symbol	Typical value*	Range considered
Landfill	L (m)	200	200-1000
	H_f (m)	1	1-10
Upper clay	D (m ² /year)	0.01	—
	n	0.4	—
	v_a (m/year)	0.0	—
Upper sand	H_1 (m)	2.0	—
	D_{H1} (m ² /year)	10.0	0.2-200
	D_{v1} (m ² /year)	0.2	—
	n	0.3	—
	v_{b1} (m/year)	1.0	0.1-1.0
	h_1 (m)	1.0	0.3-1.5
Lower clay	D (m ² /year)	0.01	—
	n	0.4	—
	v_a (m/year)	0.0	—
Lower sand	H_2 (m)	10.0	—
	D_{H2} (m ² /year)	10.0	—
	D_{v2} (m ² /year)	0.2	—
	n	0.3	—
	v_{b2} (m/year)	1.0	—
	h_2 (m)	0.3	—

* Value adopted unless otherwise noted.

low value of D_H ($D_H = 0.2$ m²/year) there is essentially no contamination upstream of the landfill. The maximum concentration in the aquifer is at the edge of the landfill ($x = 100$ m) and the concentration decreases with increasing horizontal distance away from the landfill.

By comparison, a very high dispersion coefficient ($D_{H1} = 200$ m²/year) gives rise to contamination of the aquifer upstream of the landfill. In this case the maximum concentration is not necessarily at the edge (for $t = 300$ years the maximum occurs approximately 10 m inside the landfill) and is somewhat less than that obtained for a lower value of D_{H1} . It should be noted, however, that the high value of the dispersion coefficient D_{H1} does give rise to a higher concentration at the property line (i.e. $x = 400$ m) at this

time because it allows more rapid movement of contaminant away from the landfill.

Figure 3 shows the concentration plume at one point in time. Clearly the concentration at any point will vary with time as indicated in Fig. 4 for a point in the upper aquifer beneath the edge of the landfill ($x = 100$ m) and at the property line ($x = 400$ m). At both points, the concentration increases with time until at some time t_{max} a maximum concentration is reached at that point. For subsequent times, the concentration reduces until, at very large times, the concentration becomes negligible. The magnitude and time of occurrence of this peak concentration will depend on the geometry of the problem and the soil and leachate properties. In particular, however, it depends on the height of leachate H_f .

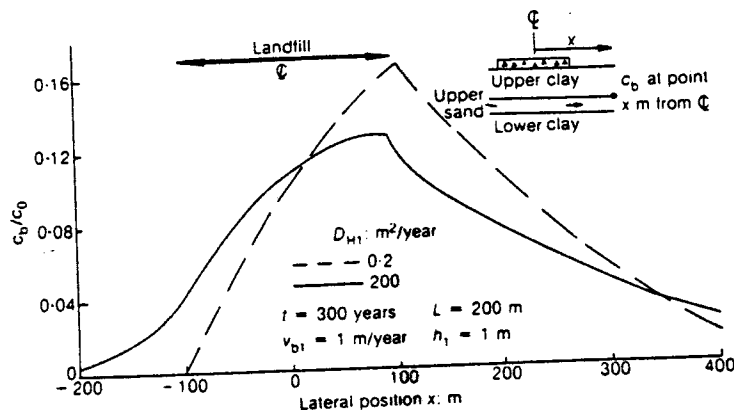


Fig. 3. Concentration plumes within the upper sand layer

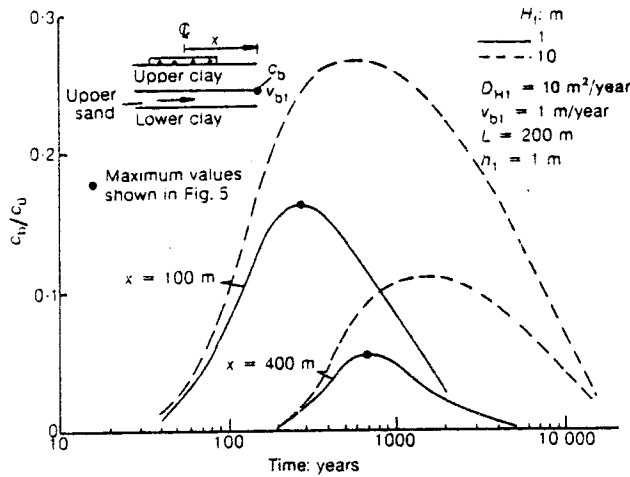


Fig. 4. Variation in concentration c_t in the upper sand layer with time for two locations ($x = 100$ m and $x = 400$ m)

The mass of contaminant contained with the landfill generally reaches a maximum value at the completion of the landfill. It may take a long time for this mass to be leached out of the solid waste; however, at some relatively short time after completion of the landfill the groundwater conditions within the landfill will generally reach a quasi-equilibrium and the concentration of contaminant within the landfill will reach a maximum value. The volume of leachate at this time may be represented in terms of the height of leachate H_f which is defined as the volume of leachate per unit plan area of the landfill.

The mass of contaminant is approximately given by the product of the volume of leachate and the maximum concentration c_0 . For a given c_0 the greater the value of H_f , the greater will be the mass of contaminant and thus the peak concentration of contaminant at any particular point in the soil will be greater. This can be appreciated by comparing the result given in Fig. 4 for $H_f = 1$ m with those presented for $H_f = 10$ m.

If the limit as H_f tends to infinity is taken (i.e. if the concentration in the landfill is assumed to remain constant at c_0 for all time) then the peak concentration at all points in the soil will eventually reach the landfill value of c_0 . This is quite unrealistic and illustrates the need to consider reasonable values of the height of leachate H_f .

Figure 3 illustrated that the horizontal coefficient of hydrodynamic dispersion D_{H1} alters the contaminant plume within the aquifer. However, under some circumstances the effect of the uncertainty regarding D_{H1} on the maximum concentration may be quite modest. For example, Fig. 5 shows the maximum base concentration and the time required to reach this maximum as a function of D_{H1} for $v_b = 1$ m/year. As can be seen, the effect of D_{H1} on the maximum concentration is greater at the edge of the landfill than at a point 300 m from the landfill. For this particular value of v_b increasing D_{H1} reduces the magnitude of the maximum concentration and increases the time required to reach this maximum. However, this is not always the case. For other values of v_b and other geometries, different trends are observed and the effect of D_H should be examined for each specific case.

If the maximum concentration at a point in the soil mass is considered it is found that the magni-

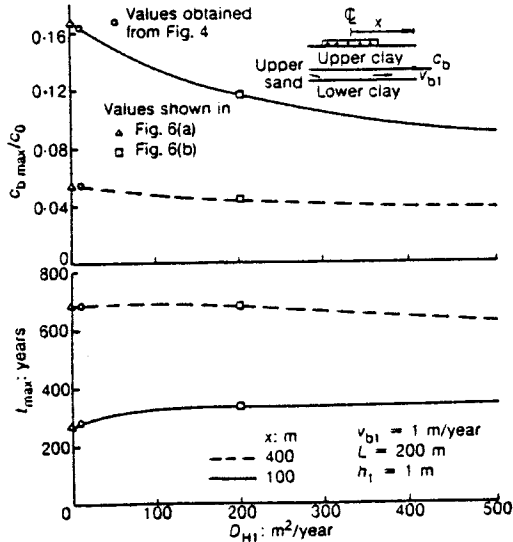


Fig. 5. Variation in maximum concentration $c_{b,max}$ in the upper sand layer with the coefficient of hydrodynamic dispersion D_{H1}

tude of this maximum value $c_{b, \max}$ (and the time t_{\max} required to reach this maximum) will depend on the horizontal advective velocity v_{b1} in the upper aquifer. For example, Fig. 6 shows the variation in this maximum value at two points in the upper aquifer ($x = 100$ m and $x = 400$ m) as a function of the assumed value of v_{b1} . An inspection of Fig. 6 reveals that the magnitude of the maximum concentration at $x = 100$ m increases for an increasing velocity v_{b1} up to a critical velocity v_{b1} of approximately 0.2 m/year and 0.6 m/year for D_{H1} values of 0.2 m²/year and 200 m²/year respectively. The magnitude of the maximum concentration becomes smaller for values of v_{b1} that are in excess of these critical velocities. Looking at the results for $x = 400$ m, a similar but more noticeable trend is observed. In this case the critical velocities are approximately 1.5 m/year and 2 m/year for D_{H1} values of 0.2 m²/year and 200 m²/year. At low velocities there is a substantial attenuation of the maximum concentration with increasing distance from the

landfill. The difference between the maximum concentration at $x = 100$ m and $x = 400$ m becomes smaller as the velocity v_b increases.

The trends observed in Fig. 6 are the result of two competing mechanisms. Firstly the longer it takes for the contaminant to move along the aquifer to the point of interest the more diffusion can occur into the adjacent clayey layers and hence the lower the concentration. The time interval between when the contaminant is released into the aquifer and when it arrives at a point outside the landfill is primarily dependent on the advective velocity v_{b1} . Thus the lower the value of v_b , the more attenuation can occur before the contaminant reaches point x . Beneath the landfill, diffusion out of the aquifer can only be down into the underlying clayey layer (at least until the maximum concentration is reached). However, at points outside the landfill diffusion can occur both upwards and downwards from the aquifer into the two adjacent clayey layers and this gives rise to the rapid attenuation in maximum concentration with distance away from the landfill.

The second mechanism is that of dilution of concentration due to mixing with water in the aquifer. The greater the velocity v_{b1} the greater will be the actual dilution of contaminant. Thus it can be seen that reducing the velocity v_{b1} will decrease the concentration by allowing more diffusion to occur, while increasing the velocity v_{b1}

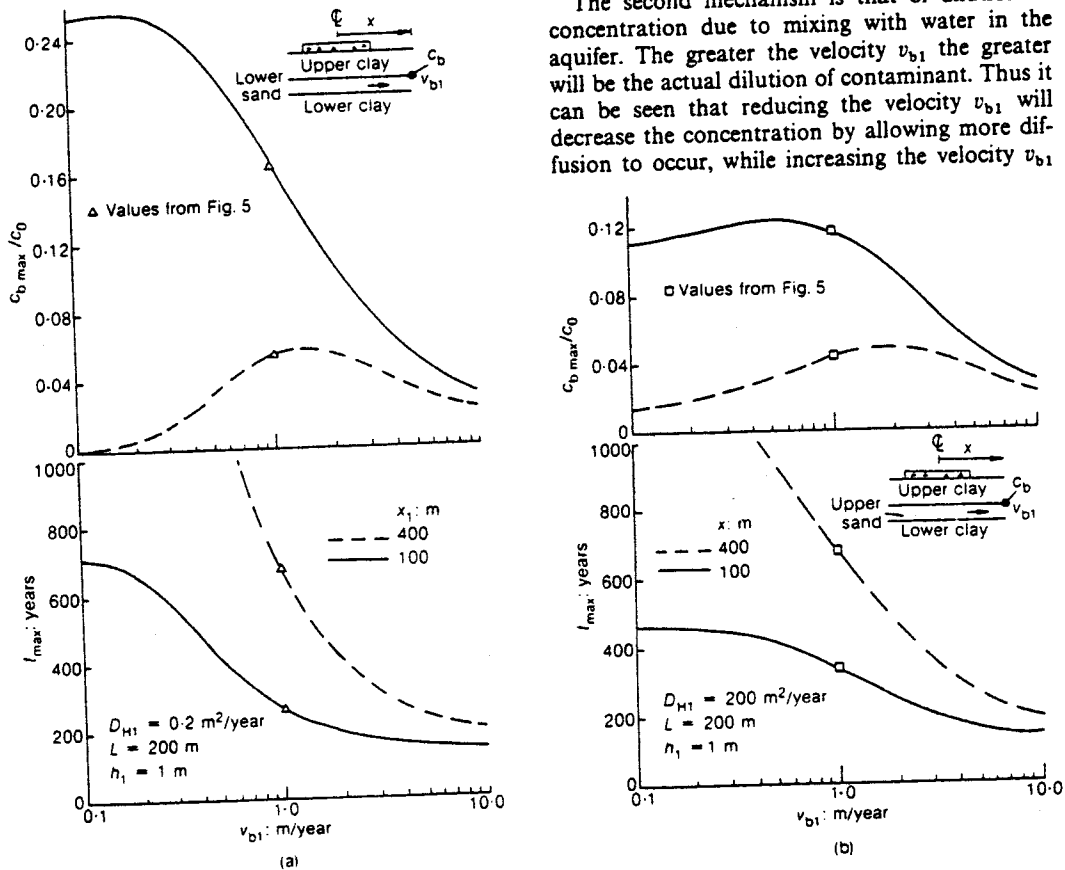


Fig. 6. Variation in maximum concentration $c_{b, \max}$ in the upper sand layer with advective velocity v_{b1} : (a) $D_{H1} = 0.2$ m²/year; (b) $D_{H1} = 200$ m²/year

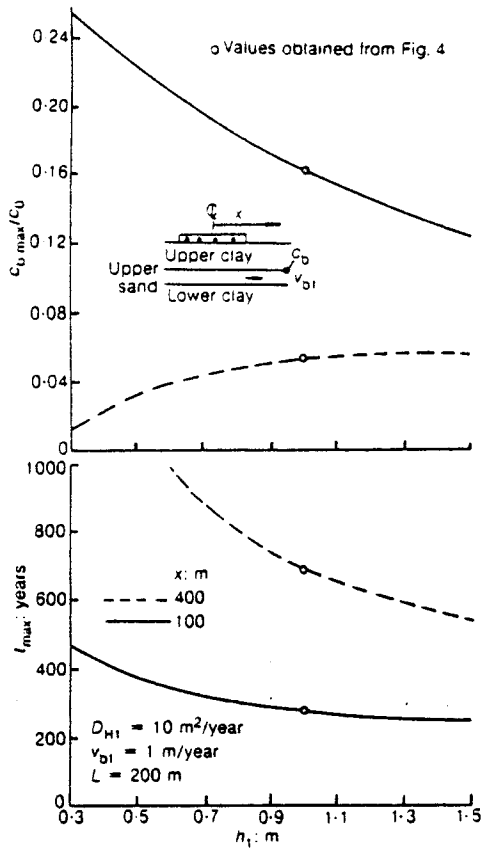


Fig. 7. Variation in maximum concentration $c_{b,max}$ in the upper sand layer with layer thickness h_1 .

will decrease the concentration by allowing more dilution. Because of these conflicting mechanisms, there is a critical velocity which results in the greatest peak concentration at a particular point.

For a given problem, the critical velocity will depend on the position x of interest and D_{H1} . This has important practical consequences since it implies that it is not necessarily conservative to design for either the maximum or the minimum expected velocity in the aquifer.

The thickness of the aquifer will have an effect on the concentration of contaminant at various locations along the aquifer. As shown in Fig. 7, the concentration of contaminant at the edge of the landfill ($x = 100$ m) tends to decrease as the thickness of the aquifer is increased. This is primarily because of an increased dilution of contaminant which occurs for large h (all other things being equal) due to the correspondingly higher flow in the aquifer. At points well away from the landfill (e.g. $x = 400$ m) the concentration of contaminant may increase with increasing thickness of aquifer (again, all other things being

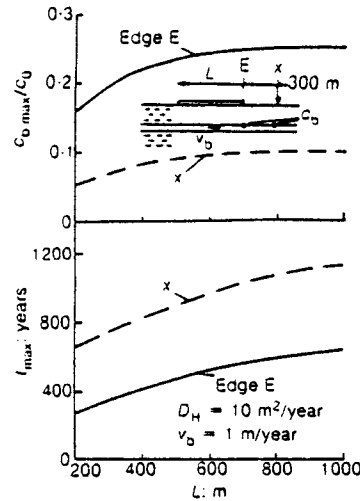


Fig. 8. Variation in maximum concentration $c_{b,max}$ in the upper sand layer with landfill width L .

equal) because the relative diffusion into the adjacent clayey soil is reduced. From a consideration of the result given in Figs 6 and 7 it can be seen that there is a fairly complex interaction between the effects of aquifer thickness, advective velocity in the aquifer and the distance to the point of interest on the maximum concentration expected to occur at that point.

All the foregoing results have been for a landfill 200 m wide (i.e. $L = 200$ m). For the other basic parameters given in Table 1, Fig. 8 shows the variation in the maximum concentration at the edge and 300 m from the edge of a landfill of variable width L . For the problem considered it can be seen that increasing the width of the landfill increases the concentration at both points of interest although the maximum concentration tends to become asymptotic to a constant value for L approaching 1000 m. The increase in concentration with L arises because of the increased mass loading of the aquifer which arises from a large total mass of contaminant within the landfill. The tendency of the asymptote to reach a constant value for very large L arises because significant diffusion can occur into the underlying clay between the time that the contaminant enters the aquifer near the upstream edge and the time that it approaches the downstream edge when the width of the landfill, L , is large. (The value of L at which this occurs will depend on v_{b1} , h , H_1 and the other parameters.)

CONCLUSION

A technique for the analysis of two-dimensional and three-dimensional pollutant

migration through a layered soil medium has been described. This formulation permits a consideration of the depletion of contaminant in the landfill with time as well as the effects of diffusion, dispersion and advection in a system involving one or more confined aquifers.

To illustrate some potential applications for the theory, consideration was given to the concentration of contaminant within a confined aquifer where attenuation could occur due to diffusion into the clayey soil above and below the aquifer. Particular attention was focused on the effects of the horizontal advection velocity and the coefficient of hydrodynamic dispersion within the aquifer together with the thickness of the aquifer. It was shown that there is a fairly complex interaction between the effect of these parameters on the maximum concentration reached at a point and it is not necessarily conservative to consider only the bounding values of the expected range of parameters when attempting to ascertain the maximum effect that the landfill will have on the water quality within the aquifer. Finally, the effect of landfill size on the concentration of particular points in the aquifer was examined.

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